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NINTH MONTHLY PROGRESS REPORT

Contract NAS 2-1460

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Code None

The effort during the reporting period February 1 to February 29, 1964 was devoted primarily to the division of the complete program into four separate jobs. The portion of the flow field computed in each job was indicated in the January progress report. As noted previously, these four jobs are on one tape and may be run consecutively.

The checkout of the new deck has been initiated and several inviscid test cases have been run with moderate success. The external flow field and most of the internal flow field was computed for the axially symmetric test case described in the fifth progress report. A two-dimensional inlet with a shock corner on the centerbody is also being run at the present time to check out the routine for intersection of shocks of the same family. The curve fitting of the external flow region is of great importance for this case, since the flow field upstream of the second shock wave is not free stream.

Due to the large number of subroutines in this program, errors of a minor nature arise quite frequently. These are being corrected as they are encountered. Now that the complete program is included in the new deck the checkout is expected to proceed at a much faster rate. The following paragraphs are devoted to a discussion of the shock boundary layer interaction procedure currently employed in the program.

Shock-Boundary Layer Interaction

The interaction between an oblique shock wave and either a laminar or turbulent boundary layer is a highly complex problem for which no exact solution exists. However, it is still possible to obtain results that are useful for engineering purposes by setting up an idealized model of the interaction and correlating the solutions with experimental data. By proper use of the conservation equations, the gross behavior of the process is examined while the minute details in the interaction region are neglected. In the present analysis, a momentum integral approach is used. A control volume of unknown length is utilized across which there is a discontinuity in pressure due to the incident and reflected shock waves. The conservation of mass and momentum equations are then applied across this control volume (Figure 2). A test for separation based on experimental data is incorporated into the program and if the pressure rise is sufficiently great, the comment "boundary layer separates" is printed out.

NASA Ames Research Center Moffett Field, Calif. ~~[illegible]~~

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Lockheed Aircraft Corp. Sunnyvale, Calif.

The validity of the analysis is greatest for thin boundary layers. If the thickness of the boundary layer is greater than the acceptable value, this is also printed out.

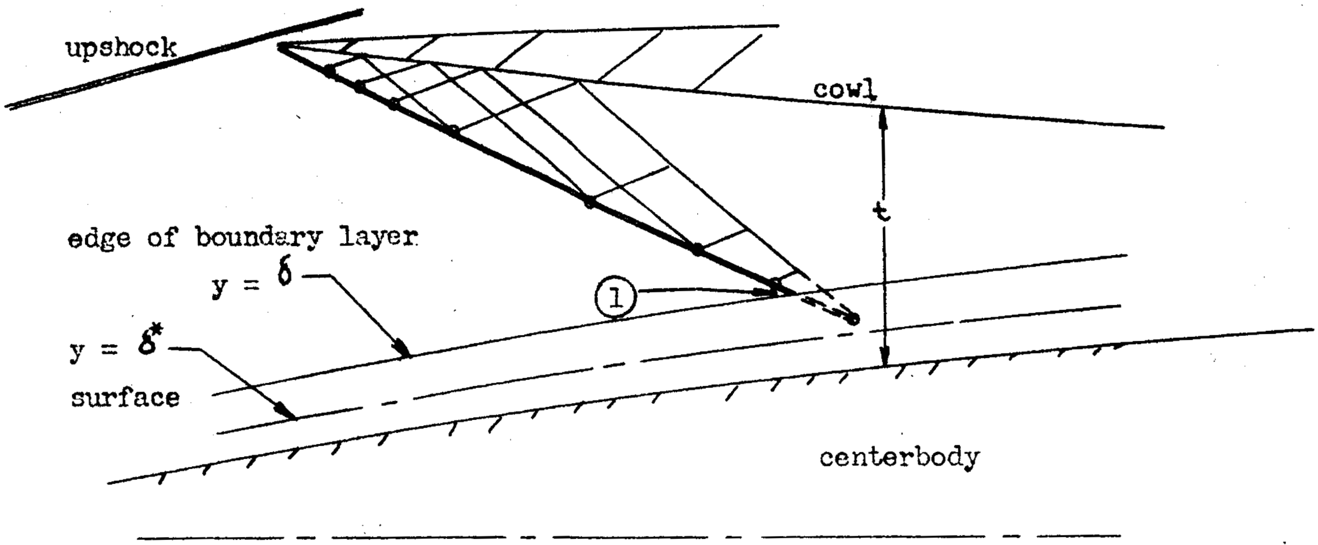

Figure 1

Computed flow field prior to shock-boundary layer interaction.

1) As the cowl lip shock is being computed, test for intersection with the edge of the boundary layer. After locating the intersection point, interpolate linearly for θ at this point (Figure 1). Region 1 properties are those immediately upstream of the intersection point.

2) Point 1 inviscid properties are obtained from a curve fit of the region and from the R-gas program. Required boundary layer properties are δ_1, δ_1^*, θ_1 and c_f. In addition, β, is needed for a laminar boundary layer. If the boundary layer is turbulent, m_1 is required.

3) Calculate δ_1/t where t is the height of the inlet duct at the station where the shock impingement occurs (Figure 1). If $\delta_1/t < 0.2$, proceed with the calculation. If $\delta_1/t > 0.2$, print out "thick boundary layer" and continue.

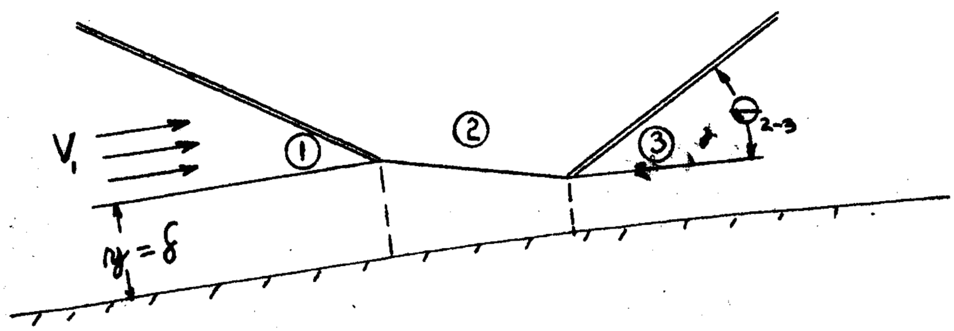

Figure 2

Control volume and designation of regions separated by incident and reflected shocks.

4) Compute P_{e_2} from equation:

$$\frac{P_{e_2}}{P_{e_1}} = \frac{2\gamma M_{e_1}^2 \sin^2\theta_{1-2} - (\gamma-1)}{\gamma+1}$$

(θ is relative to δ_{e_1})

Note: Steps 4 through 11 are only valid for a perfect gas. The modifications for a real gas are slight and have been omitted, since the procedure is essentially the same.

5) Compute ρ_{e_2} from equation:

$$\frac{\rho_{e_2}}{\rho_{e_1}} = \frac{(\gamma+1) M_{e_1}^2 \sin^2\theta_{1-2}}{(\gamma-1) M_{e_1}^2 + 2}$$

6) Find S_{e_2}, T_{e_2}, h_{e_2} and a_{e_2} from Ames R-gas program for a perfect gas. Enter with P_{e_2} and ρ_{e_2}.

7) Compute δ_{e_2} from equation:

$$\tan^2(\delta_{e_2} - \delta_{e_1}) = \left[\frac{\frac{P_{e_2}}{P_{e_1}} - 1}{\gamma M_{e_1}^2 - \frac{P_{e_2}}{P_{e_1}} + 1}\right]\left[\frac{2\gamma M_{e_1}^2 - (\gamma-1) - (\gamma+1)\frac{P_{e_2}}{P_{e_1}}}{(\gamma+1)\frac{P_{e_2}}{P_{e_1}} + (\gamma-1)}\right]$$

(Take negative sign for reflected left running shock as shown in Figure 2)

8) Compute V_{e_2} from energy equation and solve for M_{e_2} using sound speed from step 6.

$$V_e^2 = 2(h_o - h_{e_2}) + V_o^2$$

9) Compute P_{e_3} from equation:

$$\tan^2(\delta_{e_3}-\delta_{e_2}) = \left[\frac{\frac{P_{e_3}}{P_{e_2}} - 1}{\gamma M_{e_2}^2 - \frac{P_{e_3}}{P_{e_2}} + 1}\right]\left[\frac{2\gamma M_{e_2}^2 - (\gamma-1) - (\gamma+1)\frac{P_{e_3}}{P_{e_2}}}{(\gamma+1)\frac{P_{e_3}}{P_{e_2}} + (\gamma-1)}\right]$$

Where δ_{e_3} is set equal to δ_{e_1} i.e.; the reflected shock is of the same strength as the incident shock.

10) Compute θ_{2-3} from equation:

$$\sin^2 \theta_{2-3} = \frac{(\gamma+1)\frac{P_{e_3}}{P_{e_2}} + (\gamma-1)}{2\gamma M_{e_2}^2}$$

Take + sign for Figure 2 situation.

11) Compute ρ_{e_3} from equation:

$$\frac{\rho_{e_3}}{\rho_{e_2}} = \frac{(\gamma+1) M_{e_2}^2 \sin^2 \theta_{2-3}}{(\gamma-1) M_{e_2}^2 \sin^2 \theta_{2-3} + 2}$$

12) Enter Ames R gas program for a perfect gas with P_{e_3} and ρ_{e_3} to find h_{e_3}, a_{e_3}, T_{e_3} and S_{e_3}.

13) Compute V_{e_3} from equation:

$$V_{e_3}^2 = 2(h_\infty - h_{e_3}) + V_\infty^2$$

14) Compute M_{e_3} and μ_{e_3} from equations:

$$M_{e_3} = \frac{V_{e_3}}{A_{e_3}} \quad ; \quad \mu_{e_3} = \sin^{-1}\frac{1}{M_{e_3}}$$

15) Compute the following pressure coefficients:

$$C_{p_i} = \frac{P_{e_2} - P_{e_1}}{\frac{1}{2} \rho_{e_1} V_{e_1}^2}$$

$$C_{p_f} = \frac{P_{e_3} - P_{e_1}}{\frac{1}{2} \rho_{e_1} V_{e_1}^2}$$

16) Using c_{f_1} and M_{e_1}, compute G from equation:

$$G \equiv \frac{\sqrt{2 c_{f_1}}}{(M_{e_1}^2 - 1)^{1/4}}$$

17) Compute $C_{p_s} = K_s G$

Where $K_s = \begin{cases} 0.81 \text{ for laminar boundary layer} \\ 4.22 \text{ for turbulent boundary layer} \end{cases}$

18) Compare C_{p_s} from Step 22 with C_{p_f} from Step 20:

If $C_{p_f} < 2 C_{p_s}$, proceed with calculations

If $C_p > 2 C_{p_s}$, print out "Boundary Layer Separates" and continue.

19) Define the boundary layer parameters f and g as the following:

$$f = \frac{\delta^*}{\delta}$$

$$g = \frac{\theta}{\delta}$$

If boundary layer is laminar, proceed to Step 20. If boundary layer is turbulent, proceed to Step 26.

20) Assume $\beta_3^{(1)} = \beta$ of the 1st station upstream of the shock-boundary layer intersection.

21) Transfer to laminar boundary layer program and compute the properties using the assumed value of β_3. Assume an adiabatic wall (no heat transfer) and obtain other required input data from steps 11 and 12. Compute f and g from Step 19 definitions.

22) Compute g' from equation:

$$g' = \frac{1}{2}\left(\frac{\rho_{e_1} V_{e_1}^2}{\rho_{e_3} V_{e_3}^2}\right)\left(C_{p_f} - C_{p_1}\right) + \frac{\left[\frac{C_{p_1}}{2} + \left(\frac{V_{e_3}}{V_{e_1}} - 1\right)\left(1 - \frac{\delta_1^*}{\delta_1}\right) + \frac{\theta_1}{\delta_1}\right]\left[1 - f\right]}{\left(\frac{V_{e_3}}{V_{e_1}}\right)\left(1 - \frac{\delta_1^*}{\delta_1}\right)}$$

(Derived from momentum balance across control volume)

23) Assume $\beta_3^{(2)} = 0.99\ \beta_3^{(1)}$ and repeat steps 26 and 27.

24) Obtain 3rd estimate of β_3 from equation:

$$\beta_3^{(3)} = \beta_3^{(2)} - \frac{(g' - g)^{(2)}\ (\beta_3^{(2)} - \beta_3^{(1)})}{\left[(g' - g)^{(2)} - (g' - g)^{(1)}\right]}$$

Continue with Newton-Raphson procedure until

$$\left|(\beta_3)_{i+1} - (\beta_3)_i\right| \leq (\beta_3)_i \times 0.001$$

Print out final values of δ_3^*, δ_3, θ_3; c_f etc., at edge of reflected shock.

25) Locate x_3, the point of reflection of the shock from equation:

$$x_3 - x_1 = (\delta_1 - \delta_3)\ \cot\left[\left(\frac{dy}{dx}\right)_{body} - \delta_{e_2}\right]$$

where $x_3 - x_1$ is measured along the body.

Turbulent Boundary Layer

26) **Define boundary layer parameters f and g in the following way:**

$$f = \frac{\delta^*}{\delta} = \int_0^1 \left(1 - \frac{\rho u}{\rho_e u_e}\right) d\left(\frac{y}{\delta}\right)$$

$$g = \frac{\theta}{\delta} = \int_0^1 \left(1 - \frac{u}{u_e}\right) \frac{\rho u}{\rho_e u_e} d\left(\frac{y}{\delta}\right)$$

27) Obtain the values of the above parameters from the turbulent boundary layer program. They appear in equation (14) where,

$$H = \frac{\delta^*/\delta}{\theta/\delta} = \frac{\delta^*}{\theta}$$

As initial inputs, assume m_i at shock reflection point are equal to the values at incident point.

28) **Compute g' from equation:**

$$g' = \frac{1}{2}\left(\frac{\rho_{e_1} V_{e_1}^2}{\rho_{e_3} V_{e_3}^2}\right)\left(C_{p_f} - C_{p_i}\right) + \frac{\left[\frac{C_{p_i}}{2} + \left(\frac{V_{e_3}}{V_{e_1}} - 1\right)\left(1 - \frac{\delta_1^*}{\delta_1}\right) + \frac{\theta_1}{\delta_1}\right]\left[1 - f\right]}{\left(\frac{V_{e_3}}{V_{e_1}}\right)\left(1 - \frac{\delta_1^*}{\delta_1}\right)}$$

29) Assume $m_i^{(2)} = 0.99 \; m_i^{(1)}$ and repeat steps 32 and 33.

30) **Obtain 3rd estimate of m_i from equation:**

$$m_3^{(3)} = m_3^{(2)} - \frac{(g' - g)^{(2)} \left(m_3^{(2)} - m_3^{(1)}\right)}{\left[(g' - g)^{(2)} - (g' - g)^{(1)}\right]}$$

Continue with Newton-Raphson procedure until

$$\left|(m_3)_{i+1} - (m_3)_i\right| \leq (m_3)_i \times 0.001$$

31) Once final values of δ_3^*/δ_3 and θ_3/δ_3 have been obtained, compute δ_3 from equation:

$$\frac{\delta_3}{\delta_1} = \frac{\rho_{e_1} V_{e_1}}{\rho_{e_3} V_{e_3}} \left(\frac{1 - \frac{\delta_1^*}{\delta_1}}{1 - \frac{\delta_3^*}{\delta_3}} \right)$$

32) Locate x_3 as in Step 30.

33) Now that the reflection point, m and δ are known, obtain δ^*, θ, c_f, C_f, etc. from turbulent boundary layer program.

34) Return to inviscid upshock (or downshock) routine and resume computation of the flow field.

NOMENCLATURE

a	-	speed of sound
c_f	-	local skin friction coefficient
C_p	-	pressure coefficient defined by step 15
f	-	boundary layer parameter defined in step 19
g	-	boundary layer parameter defined in step 19
G	-	separation constant defined by step 16
h	-	static enthalpy
H	-	boundary layer shape factor
K_s	-	empirical factor in separation criteria
m	-	power law exponent in turbulent boundary layer solution
M	-	Mach number
P	-	pressure
S	-	local entropy
T	-	static temperature in ^{o}R
V	-	local velocity
β	-	pressure gradient parameter in laminar boundary layer
γ	-	isentropic exponent
δ	-	stream angle, boundary layer thickness
δ^*	-	boundary layer displacement thickness
θ	-	shock wave angle, boundary layer momentum thickness
ρ	-	density
μ	-	Mach angle

Subscripts

o	-	free stream conditions
1,2,3	-	regions of flow field defined in Figure 2
e	-	inviscid properties at edge of boundary layer

REFERENCES

1. Erdos, J. and Pallone, A., "Shock-Boundary Layer Interaction and Flow Separation," Avco Report RAD-TR-6123, August 1961.

2. Hakkinen, R. J., Greber, I., Trilling, L. and Arbabanel, S. S., "The Interaction of an Oblique Shock Wave with a Laminar Boundary Layer," NASA Memo 2-18-59W, March 1959.